REVIEWS

DEVELOPMENT OF BOUNDARY-LAYER THEORY IN THE USSR

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In the development of boundary-layer theory in the Soviet Union there has always been a close relationship between theory and practice. The effort to optimize the shapes of aircraft and ships and to design efficient power installations and motors has been aided significantly by progress in boundary-layer theory. It is difficult to distinguish clearly the basic stages in the development of this theory and those of its technical applications in the Soviet Union, but clearly these stages have coincided with stages in the search for better aircraft and ships, in the effort to reduce losses in machines, in the effort to intensify processes in jet engines and chemical installations, and (recently) in efforts to achieve an advanced space technology.

1. In the classical studies carried out toward the end of the last century by the prominent Russian scientists Mendeleev and Zhukovskii, one can find information, albeit qualitative, about the boundary layer and its role in hydrodynamic drag. Mendeleev [1] attributed drag to the existence of a thin layer "attached" to the surface of the object because of the viscosity of the liquid; he believed that this layer "moved and dragged along neighboring layers." In a sense anticipating the modern understanding of the drag of rough surfaces, he wrote, "...the drag of the irregularities themselves is of the same nature as the drag of a plate moving normally." Mendeleev criticized the Rankin friction theory of drag which was current at that time. As a founder of metrology, the science of measurements, he offered a profound analysis of the reasons for errors in laboratory measurements of drag.

Zhukovskii [2] offered a completely clear description of the processes occurring in a boundary layer; in his widely known series of lectures [3] he also discussed the quantitative aspect of the matter, formulating the "three-halves law" for the laminar frictional drag of a plate. The first publications on boundary-layer theory appeared in the Soviet literature in 1933-1937 [4]. The original Soviet studies dealing with boundary-layer theory contained various modifications of the Karman method [5]; new integral relations were established. The "energetic" relation proposed by Leibenzon and its generalization by Golubev [6] were among these integral relations. The first application of the theory of boundary-layer separation to explain the operation of a slotted wing appeared [7].

Empirical power-law equations for the velocities in turbulent flow in tubes were used in calculations dealing with the turbulent boundary layer for a solid of revolution [8] representing the surface of a dirigible.

Note should be taken of the interest in the heat-engineering problems of boundary-layer theory which developed in the 1930s. This interest arose with the development of methods for measuring the velocity and temperature distributions at cross sections of a boundary layer on the surface of a hot object by means of microscopic Pitot tubes, thermal anemometers, and microscopic thermocouples [9]. A variety of studies dealing with the effects of artificial boundary-layer turbulence on flow, drag, and heat exchange with the surrounding liquid [10, 11] were carried out in an effort to reduce the drag of boiler pipes and to intensify heat exchange at the surfaces of these pipes. The dependence of the heat transfer of objects, particularly spheres, on the surface distribution of lines corresponding to the conversion of laminar flow into turbulent flow (governed by the turbulence of the flow incident on the object) led to the establishment of a "thermal scale" for turbulence [12]. This scale turned out to be vastly simpler and more convenient in application than the "dynamic scale" then available, which was based on measurement of the drag of a sphere or the pressure drop between the forward and rear critical points.

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The semiempirical turbulence theories developed during this period were extended to take into account the interaction of laminar and turbulent momentum transfer in the part of the boundary layer adjacent to the surface on the nature of the over-all friction [13]. New bases were offered for the Prandtl and Karman theories [14], which at that time were clearly in need of such justification.

The prospect of flight at high supersonic velocities spurred interest in boundary-layer theory for flow in gases. Frankl' [15, 16] carried out the first Soviet studies of friction and heat transfer in laminar and turbulent boundary layers on a flat surface in a high-velocity gas flow.

2. During the prewar period (1936-1940) boundary-layer theory made progress in the Soviet Union, both in the development of calculation methods and in the development of a variety of technical applications. Experimental procedure also advanced.

The Karman method for dealing with a layer of finite thickness (at the time the only practical method for carrying out calculations for laminar and turbulent boundary layers) was refined through the use of a family of velocity profiles better reflecting the actual situation [17]. The complicated graphical method then in use to integrate the momentum equation was replaced by simpler analytic methods [18, 19].

The semiempirical Prandtl–Karman theories found wide use in the theory of a turbulent boundary layer. The logarithmic velocity profile was used to calculate the turbulent boundary layer on solids of revolution in a longitudinal flow [20]. The drag equations found were useful at the high Reynolds numbers corresponding to the actual dimensions of dirigibles and submarines. Fedyaevskii [21, 22] published a new method for calculating the turbulent boundary layer, based on the Prandtl turbulent-friction equation and approximate equations for the distribution of the "mixing path" and the frictional stress at cross sections of the boundary layer. This method became widely used to calculate the drag on wings and solids of revolution [23]. Another method for calculating the turbulent boundary layer also based on the Karman theory, was developed by Mel'nikov [24]. A long series of theoretical and experimental studies [25-27] were devoted to turbulent boundary layers on rough surfaces and to the theory of turbulent motion near such surfaces [28]. These studies furnished the basis for solving an important technical problem – increasing the speed of aircraft, ships, and submarines – by reducing the surface drag and increasing the heat transfer through the use of surface roughness.

Studies [29-32] of the effect of turbulence in the incident flow on the development of a turbulent boundary layer (in particular, on the position of the line at which the laminar boundary layer converts into a turbulent boundary layer, on the separation of the layer from the surface of the object, and on the total drag of the object) were of considerable practical importance. There was particular interest in methods for converting laboratory data on drag for use under nonlaboratory conditions (the scale effect).

There was a continuation and strengthening of the relationship between boundary-layer theory and problems of heat transfer at hot surfaces which then confronted the designers of new types of single-pass steam generators. Kruzhilin [33] introduced the concept of a temperature boundary layer of finite thickness and used this concept to calculate the heat transfer at cylindrical objects in planar flows. A new integral heat relation was established on the basis of a method very similar to the Karman method, and the solutions found by this method were used to obtain general dependences of the Nusselt number on the Prandtl and Reynolds numbers. These equations were used to calculate heat transfer at a plate and at the leading edge of a circular cylinder [34]; they were subsequently generalized [35] through an account of the dissipation of mechanical energy into heat at high flow velocities.

The first theoretical study of heat transfer in a turbulent boundary layer was reported by Shvab [36]. Using the familiar Reynolds analogy and specifying single-term power-law velocity and temperature distributions, he found equations for the heat transfer at a plate, a cylinder, and a solid of revolution in a smooth flow. In [37] he took into account the sensitivity of heat transfer to the position of the line at which the laminar boundary layer converts into a turbulent layer. In addition to having obvious technical applications, this work led to the optimum shape for the thermal turbulence gage on which he based a new thermal scale for turbulence [38].

Fedyaevskii [39] derived a semiempirical method for calculating heat transfer based on the theory of a turbulent boundary layer [21, 22]. A characteristic feature of this study was the establishment of a relationship between the Nusselt number on the one hand and the Prandtl number and the drag coefficient on the other, to replace the commonly used Reynolds number.

One of the most important achievements in the prewar development of boundary-layer theory may have been the derivation by Frankl' [40] of a new method for calculating the turbulent boundary layer at a plate in a high-velocity gas flow. This method, based on the Karman semiempirical theory of turbulence, was for a long time the only method available in the literature. Its results were confirmed very satisfactorily by subsequent experiments in flows at supersonic velocities.

The uncertainty regarding the specific properties of the boundary layer in a high-velocity gas flow spurred a study by Kibel' [41], who was the first to deal with the effect of the radiation of a flat surface on the surface temperature in a supersonic flow.

In addition to studying the boundary layer itself, Soviet scientists also dealt with the theory of turbulent streams and wakes behind objects. A theory of turbulent streams and their various technical applications was derived and studied in detail by Abramovich [42], who subsequently published the first fundamental monograph on this topic. Trubchikov studied the temperature distribution in the wake behind a hot object [43]; the results of this theory were used to work out a thermal method for measuring the turbulence in aerodynamic tubes. Petrov and Shteinberg [44] studied the frequencies of the pressure and velocity pulsations in a turbulent flow in the near and far wakes behind objects of various shapes. They showed that the familiar constancy of the Strouhal number at sufficiently high Reynolds numbers for flow around objects of similar shape also held for objects of different shape and in different orientations with respect to the flow (e.g., for the case of a wing at various angles of attack) if the characteristic linear dimension used in the determination of the Strouhal number was taken to be some average width of the wake.

There is also much applied research which deserves mention. The interference of the wing and fuselage turned interest to the behavior of a boundary layer near the intersection of two planes at a right [45] or acute [46] angle. These studies were generalized to the case of the intersection of two nonplanar surfaces [47, 48]. A related study involved a linearized formulation of the problem of a three-dimensional laminar boundary layer near the lateral edge of a plate [49]. Kalikhman [50] derived integral relations for the general case of a three-dimensional boundary layer.

Equations for the drag of a rough surface were used to derive a new equation [51] for converting laboratory data on the losses in the working rings of hydroturbines for use under nonlaboratory conditions.

Theoretical and experimental studies were made of how the lift of an aircraft wing is affected by drawing off or blowing off the boundary layer [52, 53]; this topic, new at the time, was of considerable practical importance. During the prewar years Soviet scientists developed experimental techniques for measuring the local characteristics of boundary layers under both laboratory and nonlaboratory conditions [54, 56].

In [57] the basic results of Soviet theoretical and experimental boundary-layer research were summarized in a Russian-language monograph for the first time.

3. The Second World War confronted Soviet technology, primarily aviation, with the problem of increasing the speed, altitude, and range of aircraft. Boundary-layer theory played a basic role in these studies, since though the speeds achievable at that time were far below the speed of sound, compressibility was nevertheless an important consideration. An effort was made to find better airfoil shapes for high-speed aircraft, particularly to increase the speed achievable with existing reciprocating engines.

The theoretical research [58] was supplemented by an effort to simplify and refine approximate methods for calculating boundary layers; in particular an effort was made to determine more accurately the separation point, a question of importance not only for determining the capability of a wing, but also for calculating the transition of the laminar layer into a turbulent layer. In addition to the new, purely intuitive, methods [59-61], a more rational approach developed, based on the use of exact solutions of particular problems from the theory of a laminar boundary layer for the general case of an arbitrary external-velocity distribution. The single-parameter method derived by Kochin and Loitsyanskii [62] was devoted to this problem. It was also important to derive a simple practical method for calculating a turbulent boundary layer; this problem was solved by Loitsyanskii [63]. The empirical method which he proposed was based on the similarity between the characteristics of laminar and turbulent boundary layers. This method found wide use in practical calculations of the drag profile of wings and turbine blades. Other methods based on semiempirical considerations and involving more complicated calculations were also proposed.

An important contribution was made to the theory of the boundary layer in a high-velocity gas flow by Dorodnitsyn's derivation of an integral transformation [66-68] which could be used to convert the left sides of the dynamic and thermal boundary-layer equations to the forms of the corresponding equations in an incompressible fluid. In certain particular cases (e.g., that in which the Prandtl number is unity and the surface of the object is thermally insulated), the conversion becomes exact. Dorodnitsyn's transformation immediately found wide applications and spurred a long list of Soviet studies [69, 70] of both laminar and turbulent boundary layers. Dorodnitsyn and Loitsyanskii [71, 72] derived a method for calculating a laminar boundary layer which was simple but useful only at comparatively low Mach numbers, no greater than two or three. This method is based on the same idea as the single-parameter methods mentioned above for boundary-layer theory in an incompressible fluid. It has led to equations convenient for practical applications, in particular, for determining the effect of the Mach number on the separation point on the profile of a wing in a subsonic flow. This determination was aided considerably by the simple practical methods available at that time for calculating the velocity at the outer boundary of the boundary layer on a wing in an incompressible fluid [73-75] and in a subsonic flow [76].

These papers also described a simple method for determining the point at which the laminar boundary layer converts into a turbulent layer. In this method, based on the familiar Taylor arguments about the relationship between the conversion points and the layer separation, both the effect of the wing shape and the turbulence of the incident flow are taken into account. The results furnished the basis for the rational design of wings for new high-speed aircraft. When the wing shape chosen is such that the point at which the laminar motion converts into turbulent motion is shifted downstream, and when the effect of the air compressibility is minimized, the wing drag can be reduced considerably, so the speed can be increased considerably. The use of sweptback wings in aircraft has spurred development of boundary-layer theory on corresponding surfaces [77, 78].

Effords to reduce the wing drag of aircraft and the drag of ship hulls led to attempts to solve this problem by introducing into the boundary layer a liquid or gas of lower density than that of the incident flow [79, 80].

Many studies of applied interest, carried out in aerodynamic laboratories and under flight conditions in the Soviet Union during the Second World War, are not amenable to calculation and cannot be discussed in this review. Many of these studies dealt with phenomena within the boundary layer on a wing or fuselage and with the quantitative relationship between these phenomena and the basic flight characteristics of aircraft.

4. After the war, theoretical studies continued for general problems of liquid and gas flows. New fields arose, due primarily to the increasing airspeeds and the appearance of the jet engine.

Working from the familiar mathematical concept of the moment of a function, Loitsyanskii [81] introduced a new system of integral relations – moment equations – and used them to derive a method of calculating a planar and laminar boundary layer, which subsequently also found use in heat calculations [82, 83]. The moment method was used in problems involving both heat and mass transfer in planar boundary layers at porous walls in a series of articles by Shul'man [84-86].

Interest developed in problems involving three-dimensional boundary layers. Bogdanova [87] derived a new class of exact solutions of the equations of a three-dimensional laminar layer; these exact solutions were used to construct an approximate single-parameter method for solving problems with an arbitrary external velocity distribution.

Turbine technology spurred the solution of many dynamic and heat problems dealing with the calculation of laminar and turbulent, velocity and temperature, three-dimensional boundary layers on rotating discs and other axisymmetric objects.

These solutions and their analysis formed the content of a special monograph by Dorfman [88], which dealt with both steady-state and nonsteady-state boundary layers on rotating objects. Dolidze [89] pointed out a rigorous method for solving the equations of a nonsteady-state boundary layer on a rotating disc for an arbitrary time dependence of the angular velocity. The exact solution was found for this problem for an arbitrary power-law angular velocity by Rozin [90].

A series of studies carried out in boundary-layer theory during the postwar period at the Central Aerodynamic Institute were reported in [91]. Dorodnitsyn [92] published in final form his theory for laminar boundary layer in a high-velocity gas flow. Struminskii [93] published a theory for a three-dimensional boundary layer on a wing with slip in the flow of an incompressible liquid or gas. Sokolova studied the effect of radiation on the surface temperature of a plate [94] and a cone [95] in supersonic flows at high Mach numbers. Struminskii [96] generalized the Karman–Polhausen method to the case of a nonsteady-state boundary layer in an incompressible fluid on a cylinder and on a solid of revolution. He found an exact solution of the equations for a nonsteady-state boundary layer for the case of longitudinal flow around a plate.

Simple methods for calculating nonsteady-state laminar boundary layers were suggested by Rozin [97] and Dobryshman [98]. Dobryshman generalized the method of Shvets [99], proposed for a steady-state boundary layer, to the nonsteady-state case. Targ took a similar approach, based on his own method for calculating a steady-state layer. He reported the solution of certain classes of problems in the dynamics of a viscous, incompressible fluid, including boundary-layer problems [100].

The adoption of the jet engine in aviation considerably spurred the development of boundary-layer theory in the postwar period. Boundary-layer theory furnished a method for calculating the loss in a planar array of airfoils simulating the working rings and guide apparatus of turbines and compressors used in turbojet engines; this theory yielded expressions for the propagation of the jet stream in the surrounding air.

Loitsyanskii's method for calculating the loss and efficiency of planar turbine and compressor arrays in separationless-flow operation [101-103] turned out to be quite satisfactory and was adopted for engineering calculations [104]. The basic equation recommended in this method is a generalization of the familiar Squire–Young equation for an isolated airfoil to the case of an array of airfoils.

Because of the wide use of jets in aviation and rocket technology, in gas boilers, and in chemical apparatus, the theory of jets has been developed extensively in the Soviet literature. Soviet scientists were the first to formulate theoretically and solve problems dealing with jets propagating along solid surfaces [105, 106], twisted jets, and fan-shaped jets [107, 108]; they generalized the results found in the classical theory of free jets to many more complicated cases which arise in technical applications: jets in wakes, jets in media having varying physical constants, jets in inert gases and in gases reacting chemically with their surroundings, burning jets, etc. Here we cannot dwell on the development of this area of over-all boundary-layer theory, which long ago became an independent branch. For a detailed discussion of Soviet achievements in the theory and technical applications of jets the reader is directed to the basic monograph by Abramovich [109] and to a collection of original papers edited by Vulis [110]; this collection also contains an exhaustive bibliography.

A recent monograph by Ginevskii [111] dealt with applications of integral methods of boundary-layer theory in the theory of turbulent jets and wakes. Results obtained by Soviet and foreign scientists were reported; the emphasis was on the results obtained by Ginevskii and his colleagues regarding jets and wakes in both compressible and incompressible fluids, particularly nonisothermal jets in homogeneous gases and gaseous mixtures.

5. The last decade, which can fairly be called the beginning of the space age, saw rapid development of boundary-layer theory in the USSR. The problems arising in the aerodynamics and thermodynamics of space flight have been very complicated, both physically and mathematically. The development of high-speed digital computers, whose sophistication is advancing daily, has allowed scientists to cope with the integration of the differential equations of boundary-layer theory, despite the fact that the classical Prandtl equation has long since given way to a very complicated system of differential equations containing many unknown functions characterizing the variety of gas dynamic and thermodynamic processes in boundary layers at hypersonic gas velocities.

New difficulties have arisen because physics research has not yet produced models (sufficiently simple for practical use) for energy and mass transfer in high-speed flows of a homogeneous gas or a mixture of inert and reacting gases. Investigators do not know the pertinent coefficients for this type of transfer or the behavior of these coefficients as functions of the gas-dynamic and thermodynamic parameters of the flow. A particular difficulty has been the need to analyze transient transfer processes in a boundary layer in a thermodynamically nonequilibrium situation, because of the high flow velocities. This complexity is reflected in the variety of relaxation times for the distribution of molecular kinetic energy with respect to the internal molecular degrees of freedom and in the relaxation nature of molecular dissociation, molecular ionization, and radiation.

This set of complex problems, along with the problems which arose during previous periods and which remain important, are still being studied in the development of boundary-layer theory in the Soviet Union.

A problem as yet unresolved and requiring analysis is that of the steady-state three-dimensional boundary layer; in particular, it is necessary to determine the mechanism for three-dimensional separation of laminar and turbulent boundary layers on the surface of a cone at an arbitrary angle of attack in a gas flow [112-114] and to study the interaction of the separation with a condensation discontinuity [115].

There has been a continuing study of transient boundary layers in high-velocity flows; new classes of self-similar solutions have been found for both planar boundary layers and those on solids of revolution [116]; and transient-mixing problems have been formulated and solved for turbulent jets [117, 118] and aerodynamic wakes. In a series of studies by Oleinik important existence theorems have been established for the solutions of equations for steady-state and nonsteady-state boundary layers (see the basic paper [119], which contains a detailed bibliography).

Great efforts have been made to develop various methods for approximating laminar and turbulent boundary layers in high-velocity gas flows for the cases in which there is heat transfer and an arbitrary pressure distribution along the surface [120]. A variety of applications of semiempirical turbulence theories to problems of this type were reported by Ginzburg et al. [121-123]. A problem related to space technology – the cooling of a solid surface in a hypersonic flow – led to interest in the effect on heat transfer of the blowing of air or some other gas into a boundary layer through a porous surface. There have been a vast number of Soviet theoretical and experimental studies on this topic; among the first Soviet papers published in this field are those by Mugalev [124, 125], Motulevich [126-128], Lapin [129-131], and Avduevskii and Obroshova [132].

Computers have been used in boundary-layer calculations based on the Dorodnitsyn integral-relation method [133] and on the basis of ordinary finite-difference methods. Paskonov [134] published a standard program for computer integration of the differential equations of a laminar boundary layer in a high-velocity gas flow. Applications of finite-difference methods for various problems involving a laminar boundary layer in a gas were reported by Petukhov [135], Brailovskaya and Chudov [136] (on steady-state layers), and Paskonov and Ryabin'kina [137] (on nonsteady-state layers). Specific features of the boundary layer in multicomponent gas mixtures were studied numerically by Bulatskaya [138] and Anfimov [139]. Similar methods have been used in problems dealing with boundary layers for the cases in which air is pumped in or drawn off [140].

Numerical methods have been used to study a variety of three-dimensional problems involving a laminar boundary layer in a supersonic gas flow: on an infinitely long elliptical cylinder with slip [141], on an ellipsoid of revolution at some angle of attack [142], on a spherical segment [143], etc.

The new "parametric method" of Loitsyanskii [144-147] occupies an intermediate position between the exact numerical methods and approximate analytic methods. It is based on a "universal" form of the differential equations for a planar boundary layer, i.e., one which does not depend on the characteristics of the particular problem (the distributions of the external velocity, temperature, etc.). After these universal equations have been integrated numerically once, they can be used to solve particular problems. Tables can be used to find the relationships between the basic characteristics of the boundary layer and the form parameters which appear as independent variables in the universal equations. The parametric method has been successfully used in problems involving a steady-state laminar boundary layer – three-dimensional layers in incompressible fluids [148], planar layers in high-velocity flows of homogeneous gases [149, 150] and gases at a dissociative equilibrium [151, 152], and planar [153, 154] and three-dimensional [155, 156] boundary layers in electrically conducting fluids in the presence of a magnetic field. The method can be formally extended to the case of a turbulent boundary layer [142].

The rapid convective and radiative gas heating which occurs in a hypersonic boundary layer has several unique effects: dissociation of the gas, its ionization and melting and subsequent evaporation (or direct sublimation) of the solid surface. As a result, foreign gases enter the boundary layer, changing the flow into a multicomponent flow and complicating the description of transfer processes; sometimes there are also gases present which react chemically.

In the first studies reported (in the early 1960s) by Soviet scientists in this field, which is of extreme importance for space technology, the formation kinetics of the new components of a gaseous mixture was not taken up, and only the simplest aspects of dissociation and ionization were analyzed: the "frozen" and "equilibrium" processes. These topics were treated in [116, 130, 133, 146, 147]; a laminar boundary layer was treated in [157] and a turbulent boundary layer was treated in [158-163].

Subsequent research dealt with all of these complex processes, and account was taken of the thermodynamically nonequilibrium nature of the transfer processes characteristic of the aerodynamics of hypersonic boundary layers.

The problem of returning space vehicles into the dense atomosphere of the earth at velocities on the order of the second cosmic velocity presented boundary-layer theory with the problem of thermally insulating the surface of the vehicle. We omit a discussion of the choice of materials for covering the vehicle and of the complicated processes by which these coverings are destroyed (with which, of course, Soviet specialists were forced to deal); here we will discuss only the basic research directly related to the pertinent boundary-layer theory.

The first Soviet theoretical studies contained quantitative calculations of the surface temperature and mass-removal rate based on simplified models for the physicochemical processes at the surface and a "frozen" boundary layer [160-162]. In [160, 161] the simplest models were treated – the "boiling" of solid carbon dioxide and the equilibrium combustion of graphite in pure oxygen.

The fundamental study by Tirskii [163] yielded an exact solution of the problems of the equilibrium and nonequilibrium sublimation of a blunt object near the critical point for an arbitrary temperature dependence of the object's physical properties.

In many subsequent Soviet studies this topic was developed and a deeper understanding was gained of the variety of processes occurring in the gas flow and in the solid itself which are responsible for the destruction of the object [165-167]. To simplify practical calculations, Tirskii [168-170] used a generalized analogy between heat and mass transfer in a multicomponent boundary layer valid near the frontal critical point of the object. The subsequent development involved generalization of the method to the case of an arbitrary pressure distribution along a laminar boundary layer and to the case of a turbulent boundary layer [171-175]. The corresponding thermodynamic processes were also taken into account.

At present the understanding of the destruction of solid surfaces in hypersonic gas flows has advanced to the point [176, 177] at which destruction problems can be handled by computers for specific objects, and the entire set of physicochemical reactions, involving the dissociation and ionization of a gas mixture in the boundary layer and the effect of radiation, can be taken into account [178-180].

Soviet scientists have been particularly concerned with the effect of the thermodynamically nonequilibrium nature of heat transfer, dissociation, and ionization in a hypersonic boundary layer on heat transfer at the surface. Studies have been made of the relaxation of the distribution of kinetic energy with respect to internal molecular degrees of freedom [181-184], of the relationship between vibrational and dissociative relaxation, and of the effects of these types of relaxation on heat transfer in a laminar boundary layer [185, 186].

The engineering problem of producing a magnetohydrodynamic generator has encouraged interest in magnetohydrodynamics in general and in the theory of the boundary layer in an electrically conducting fluid or in an ionized gas (plasma), in particular. Several early studies [192, 193] dealt with the basic formulation of the boundary-layer problem in magnetohydrodynamics; subsequent studies have dealt with particular problems, e.g., the rotation of a disc in a viscous conducting fluid in the presence of a magnetic field [194-196], and the boundary layer formed at the walls of the magnetohydrogynamic duct [197, 198].

The motion of non-Newtonian fluids has come under study in connection with the development of new materials (plastics and polymers) in petroleum technology. The group headed by Academician A. V. Lykov made an important contribution to the theory of boundary layers in non-Newtonian fluids. A series of publications by this group [187-191] described the results of theoretical and experimental studies of friction and heat and mass transfer in laminar boundary layers of nonlinearly viscous non-Newtonian media. The existing generalizations of the solutions of boundary-layer problems to the case of non-Newtonian fluids were reviewed in [199]. Note should be taken of the possible link between this field of boundary-layer theory and magnetohydrodynamics [200].

Questions involved in boundary-layer theory for incompressible fluids and some of the new results cited above were discussed in [201]. In addition, boundary-layer theory has received considerable attention in general courses in fluid mechanics [202, 203].

6. We conclude this review by taking up two topics which in a sense occupy an intermediate position between the classical Prandtl boundary-layer theory and the general dynamics of a viscous gas described

by the Navier–Stokes equations. These are the interaction of a laminar boundary layer with an external, inviscid hypersonic flow and the problem of a boundary layer in a gas for the case of closed or open separation zones.

In the upper atmosphere, where the Reynolds numbers decrease, and at high speeds, at which the Mach numbers increase, the laminar boundary layer becomes so thick that the classical Prandtl boundary-layer equations and external boundary conditions neglecting the reciprocal effect of the boundary layer on the external, inviscid hypersonic flow, cannot be used. In these situations we must resort to approximations in the solutions of the Navier–Stokes equations which are more accurate than the Prandtl approximation, or we must even try to solve these equations directly.

The first Soviet studies in this field were concerned with establishing the general similarity criteria for this type of planar and axisymmetric flow and with finding approximate solutions of the generalized equations of a laminar boundary layer for thin, pointed objects. Analyses were made of both the weak interaction, corresponding to moderate Mach numbers and relatively high Reynolds numbers [204-206], and the strong interaction, with very large Mach numbers and finite Reynolds numbers [207-212]. In many of the strong-interaction studies, the low density of the gas and the particular boundary conditions at the wall (the slip of the gas and the temperature discontinuity) have been taken into account. The solution has been written as a power series in powers of a small parameter and ordinarily only the first approximation has been used. Matveeva and Sychev [213] have attempted to refine this analysis.

Ladyzhenskii [214-216] treated three-dimensional flow (a triangular plate; a thin object at a small angle of attack).

It is very difficult to treat cases which lie between the cases of strong and weak interactions, particularly those involving an interaction which varies in intensity along the boundary layer, as is observed in the hypersonic flow around thin but blunt objects. In this case, the velocity and temperature boundary layers are accompanied by an entropy layer, in which the entropy of the gas changes sharply along the direction normal to the surface. The eddy motion which arises in the outer inviscid flow under these conditions further complicates the problem [217]. To avoid the serious difficulties which arise in attempts to join the solutions of the boundary-layer equations found in the approximations following the Prandtl approximation with the solutions found for the external inviscid flow, attempts have been made to solve the inverse problem: to construct the flow around some thin, blunt object whose contour is governed by the gas flow behind a shock wave with a specified front shape (e.g., parabolic) [218].

The serious calculation difficulties which arise in this type of approximate solution have forced investigators to turn to a direct numerical solution of the Navier–Stokes equations [219-222]. As was shown in [223, 224], the difficulty associated with the indeterminacy of the boundary condition at infinity in the wake region can be avoided by exploiting the limiting case of an infinitely large Mach number in the incident flow.

Two monographs are now available dealing with the theory of the interaction of the boundary layer with the external inviscid flow [225, 226].

Theory of the motion of liquids and gases near the separation of a boundary layer and in separated zones also goes beyond the classical Prandtl theory and requires the use of either higher approximations or a direct integration of the Navier–Stokes equations. The basic goal of this theory has been to calculate the pressure drag of an object in supersonic flight; the main reason for this theory is the bottom pressure behind the stern section of the object.

The first simplified models for separated flow, based on semiempirical theories for the mixing of a homogeneous gas flow with an adjacent gas at rest, were originally used to study one aspect of the situation in the separation zone – determination of the mass of the gas drawn into the mixing zone [227-230]. The return flow in the separation zone has also been treated [231, 232].

If we treat the separation line as fixed and treat the mass of the gas in the separation zone as constant, we can determine the characteristics of the flow from the separation zone, the zone configuration, and the bottom pressure. Such calculations have been carried out in axisymmetric flows behind projections [233-236] and behind objects having simple shapes [227, 237-239]. The formation of a separation zone at the front of a blunt solid of revolution caused by a needle inserted upstream was studied in [229, 230, 240, 241].

If, on the other hand, the position of the separation line is not known beforehand, it is considerably more difficult to calculate the cutoff flow; experimental data and similarity considerations must be used [242, 243]. Methods for calculating the cutoff zones behind an acute angle and behind a sphere have also been developed which are not directly based on experiment [228, 242, 243]. The heat fluxes to laminar and turbulent separation zones have been evaluated. A study has been made of the time required for the establishment of steady-state flow in a turbulent bottom zone [244].

In the case of relatively small, closed separation zones, a calculation can be carried out over the entire flow region by using the ordinary boundary-layer equations and inserting a correction for the inverse effect of the boundary layer on the inviscid flow, by taking the displacement thickness into account. El'kin and Neiland [228] used integral conditions on the momenta and enthalpy to solve this type of problem; they substituted single-parameter velocity and enthalpy families into the boundary-layer cross sections.

A topic somewhat removed from the mainstream of the development of boundary-layer theory is the "inverse transition" of a turbulent boundary layer into a laminar layer [245-248]. This transition usually occurs when there is a negative velocity drop in convergent channels and in converging regions of a boundary layer in gas flows (especially at near-sonic velocities). There is much interest in the new studies by Bulakh [249] of the behavior of the boundary layer near a corner on an object.

We could not end a review of such a rapidly developing field of fluid mechanics as modern boundary-layer theory without pointing out that Soviet scientists are continuing their study of the physical aspects of hypersonic flow in boundary layers. They are, of course, primarily interested in radiation absorption in gaseous mixtures and in determining the radiative flux toward the surface of an object, taking into account the selectivity of the radiation and its interaction with other processes in the boundary layers. The more classical aspects of boundary-layer theory (involving the future development of methods for solving three-dimensional problems; the study of transient flow, magnetohydrodynamic boundary layers, and boundary layers in non-Newtonian fluids; and many other new questions) of course remain important.

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